

PEMBAHASAN OLIMPIADE MATEMATIKA

TINGKAT SMA/MA

A. Pilihan Ganda

$$1. \sqrt[4]{49 + 20\sqrt{6}} = \sqrt{\sqrt{49 + 2\sqrt{600}}} = \sqrt{5 - 2\sqrt{6}} = \sqrt{3} + \sqrt{2}$$

Jawaban : B

$$2. \text{Syarat akar - akar real : } D \geq 0 \Leftrightarrow p^2 - 4q \geq 0$$

$$(p - 2\sqrt{q})(p + 2\sqrt{q}) \geq 0$$

$$p \leq -2\sqrt{q} \text{ (tidak mungkin) atau}$$

$$p \geq 2\sqrt{q} \text{ (mungkin)}$$

q	p	Banyak Persamaan
1	2,3,4,5,6	5
2	3,4,5,6	4
3	4,5,6	3
4	4,5,6	3
5	5,6	3
6	5,6	3

Jadi, banyak persamaan adalah 19 buah

Jawaban : E

$$3. \log 6^{50} = 50 \log 6 = 50(\log 3 + \log 2) = 50(0,4771 + 0,3010) = 38,9050$$

Hal ini berarti : $38 < \log 6^{50} < 39$

$$\log 10^{38} < \log 6^{50} < \log 10^{39}$$

$$10^{38} < 6^{50} < 10^{39}$$

Jadi, banyaknya angka dari 6^{50} adalah 39 buah

Jawaban : D

$$4. \text{Urutan yang mungkin} = 4!3! - 3!2! = 72 \text{ cara.}$$

Jawabn : C

$$5. \text{Dituliskan } f(x) = \cos(x + \alpha) + \cos(x + \beta) + \cos(x + \gamma) \text{ dan } f(x) \equiv 0 \text{ untuk } x \in \mathcal{R},$$

$$f(-\alpha) = f(-\beta) = f(-\gamma) = 0$$

$$\text{Maka } \cos(\beta - \alpha) + \cos(\gamma - \alpha) = -1$$

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Sehingga

$$\cos(\beta - \alpha) = \cos(\gamma - \alpha) =$$

$$\cos(\gamma - \beta) = -\frac{1}{2}$$

Karena $0 < \alpha < \beta < \gamma < 2\pi$, maka

$$\beta - \alpha, \gamma - \alpha, \gamma - \beta \in \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

$$\beta - \alpha = \gamma - \alpha = \frac{2\pi}{3}$$

$$\gamma - \beta = \frac{4\pi}{3}$$

jawaban : B

$$6. [(a + b) + (c + d)]^2 = 225$$

$$(a + b)^2 + 2(a + b)(c + d) + (c + d)^2 = 225$$

$$a^2 + b^2 + c^2 + d^2 + (a + b)(c + d) +$$

$$(a + c)(b + d) + (a + d)(b + c) = 225$$

$$a^2 + b^2 + c^2 + d^2 + 36 + 56 + 54 = 225$$

$$a^2 + b^2 + c^2 + d^2 = 79$$

Jawaban : C

$$7. \lim_{x \rightarrow 0} \frac{ax \sin x + b}{\cos x - 1} = 1$$

- $a. 0 \sin 0 + b = 0$

$$b = 0$$

- $\lim_{x \rightarrow 0} \frac{ax \sin x}{-2 \sin \frac{1}{2}x^2} = 1$

$$\frac{a \cdot 1}{-2 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 1$$

$$a = -\frac{1}{2}$$

Jawaban : A

$$8. \frac{1-3\cos x}{\sin x} + \frac{7\sin x}{1-\cos x} = 3 \csc x$$

$$\frac{1 - \cos x - 3 \cos x - 3 \cos^2 x + 7 \sin^2 x}{\sin x(1 - \cos x)}$$

$$= \frac{3}{\sin x}$$

$$10 \cos^2 x + \cos x - 5 = 0$$

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{201}}{20}$$

Sehingga ada 3 penyelesaian x memenuhi

Jawaban : D

$$9. x^2 = (\sqrt{2} + \sqrt{5})^2$$

$$x^2 = 2 + 5 + 2\sqrt{10}$$

$$(x^2 - 7)^2 = (2\sqrt{10})^2$$

$$x^4 - 14x + 49 = 40$$

$$x^4 - 14x + 9 = 0$$

Jawaban : B

$$10. 4^x = 6 \rightarrow 2^x = \sqrt{6}$$

$$\frac{8^x + 8^{-x}}{2^x + 2^{-x}} = \frac{(2^x)^3 + (2^x)^{-3}}{2^x + 2^{-x}} = (2^x)^2 - 1 +$$

$$(2^x)^{-2} = 6 - 1 + \frac{1}{6} = \frac{31}{6}$$

Jawaban : C

$$11. \frac{1}{3} AD \cdot \left(\frac{1}{2} BC \cdot AC \cdot \sin 45^\circ\right) \geq V_{D.ABC} = \frac{1}{6}$$

$$\text{Jadi } AD \cdot BC \cdot \frac{AC}{\sqrt{2}} \geq 1$$

$$AD + BC + \frac{AC}{\sqrt{2}} \geq 3 \cdot \sqrt[3]{AD \cdot BC \cdot \frac{AC}{\sqrt{2}}} \geq 3$$

Memenuhi jika dan hanya jika $AD = BC = \frac{AC}{\sqrt{2}} = 1$, $AD \perp ABC$, jadi $DC =$

$$\sqrt{AD^2 + AC^2} = \sqrt{3}$$

Jawaban : C

$$12. \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{2016^2}\right) \left(1 - \frac{1}{2017^2}\right)$$

$$= \left(\frac{1.3}{2^2}\right) \left(\frac{2.4}{3^2}\right) \left(\frac{3.5}{4^2}\right) \dots \left(\frac{2016.2018}{2017^2}\right) \left(\frac{2017.2019}{2018^2}\right)$$

$$= \frac{2019}{4036}$$

Jawaban : C

13. Misalkan akar - akar dari $2x^2 - 20x + (7k - 1) = 0$ adalah m dan n dimana $m : n = 2p : 3p$ Maka :

$$m + n = 10$$

$$5p = 10$$

$$p = 2$$

Diperoleh $m = 4, n = 6$

$$m \cdot n = \frac{7k - 1}{10}$$

$$k = 7$$

Maka : $k^2 + n^2 = 85$

Jawaban : D

$$14. (f^{-1} \circ g^{-1})(x) = 2x - 4$$

$$g \circ f(x) = \frac{x+4}{2}$$

$$g(x) = \frac{x-3}{2x+1}$$

$$\frac{f(x)-3}{2f(x)+1} = \frac{x+4}{2}$$

$$\text{Maka, } \frac{f(2)-3}{2f(2)+1} = \frac{2+4}{2}$$

$$f(2) - 3 = 6f(2) + 3$$

$$f(2) = -\frac{6}{5}$$

Jawaban : B

$$\log_x(8-x) - \frac{1}{\log_{x+1} x} = 1$$

$$\log_x(8-x) - \log_x(x+1) = 1$$

$$\log_x\left(\frac{8-x}{x+1}\right) = \log_x x$$

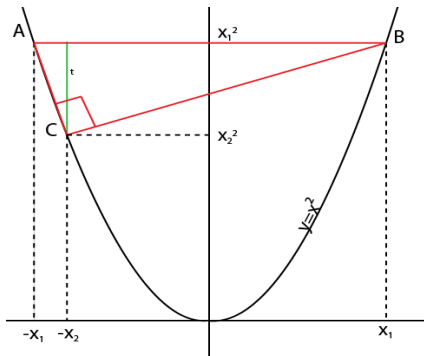
$$\frac{8-x}{x+1} = x$$

$$(x+4)(x-2) = 0$$

$$x = -4(TM) \cup x = 2(M)$$

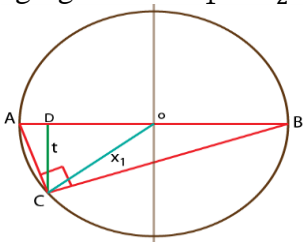
Jawaban : B

16.



Ilustrasi soal, lihat gambar.

Kita tentukan bahwa $AB = 2x_1$, dan tinggi segitiga adalah $x_1^2 - x_2^2$



$$OC^2 = DC^2 + OD^2 = (x_1^2 - x_2^2)^2 + x_2^2$$

$$= x_1^2$$

$$\sqrt{x_1^2 - x_2^2} = x_1 - x_2$$

$$x_1^2 - x_2^2 = 1$$

Selanjutnya :

$$L\Delta = 5$$

$$\frac{2x_1(x_1^2 - x_2^2)}{2} = 5$$

$$x_1 = 5$$

Ordinat titik B adalah : $y_B = x_1^2 = 25$

Jawaban : E

17. Matriks A berukuran $n \times n$ berlaku :

$$\det kA = k^n \det A, k \in \mathcal{R}$$

$$\text{Maka : } \det 2A = 2^5 \det A = 32 \cdot \left(-\frac{1}{8}\right) = -4$$

Jawaban : B

18. $1500 < 11n < 2000$ sehingga $136 < n < 182$

$970 < 7n < 1275$ sehingga $138 < n < 183$

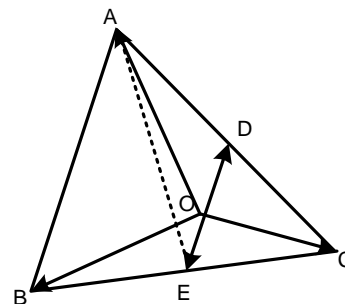
$690 < 5n < 900$ sehingga $138 < n < 180$

Maka $138 < n < 180$

Bilangan yang habis dibagi 3 dan 5 maka bilangan tersebut habis dibagi 15 yaitu 150 dan 165

Jawaban : A

19.



D dan E titik tengah dari AC dan BC

$$\vec{OA} + \vec{OC} = 2\vec{OD}$$

Dan

$$2(\vec{OB} + \vec{OC}) = 4\vec{OE}$$

$$\text{Sehingga : } \vec{OA} + 2\vec{OB} + 3\vec{OC} = 2(\vec{OD} + \vec{OE}) = 0$$

\vec{OD} dan \vec{OE} kolinier dan $|\vec{OD}| = 2|\vec{OE}|$, maka :

$$L\Delta AEC : L\Delta AOC = 3 : 2$$

Dan $L\Delta ABC : L\Delta AOC = 3 : 1$

Jawaban : C

20. Misalkan $y = \sqrt{x-3} + \sqrt{6-x}$, $3 \leq x \leq 6$

Maka $y^2 = x - 3 + 6 - x + 2\sqrt{(x-3)(6-x)} \leq 2[(x-3)(6-x)] = 6$

Jadi, $0 < y \leq \sqrt{6}$, nilai maksimum k adalah $\sqrt{6}$

Jawaban : D

21. $a^4 + a^2 - 1 = 0$

$$a^6 + a^4 - a^2 = 0$$

$$a^6 + 2a^4 - 1 = 0$$

$$a^6 + 2a^4 = 1$$

Jawaban : A

22. $3^{32} - 2^{32}$

$$= (3^{16} - 2^{16})(3^{16} + 2^{16})$$

$$= (3^8 - 2^8)(3^8 + 2^8)(3^{16} + 2^{16})$$

$$= (3^4 - 2^4)(3^4 + 2^4)(3^8 + 2^8)(3^{16} + 2^{16})$$

$$= (3^2 - 2^2)(3^2 + 2^2)(3^4 + 2^4)(3^8 + 2^8)(3^{16} + 2^{16})$$

$$= (3 - 2)(3 + 2)(3^2 + 2^2)(3^4 + 2^4)(3^8 + 2^8)(3^{16} + 2^{16})$$

$$= 1 \times 5 \times 13 \times 17 \times 401 \times (3^{16} + 2^{16})$$

Jawaban : D

23. $\frac{(x+y)(x+y)}{xy} = \frac{25}{6}$

$$(x+y)(x+y) = 25$$

$$(x+y)^2 = 25$$

$$x + y = 5$$

$$xy = 6$$

Pasangan yang mungkin (2,3) dan (3,2)

$$\text{Nilai minimum } (x-y)^3 = (2-3)^3 = -1$$

Jawaban : E

24. $C_2^{2016} = 2.028.096$

Jawaban : A

25. $64^{14} \times 5^{80} = (2^6)^{14} \times 5^{80}$
 $= 2^{84} \times 5^{80}$

$$= 2^4 \cdot 2^{80} \times 5^{80}$$

$$= 16 \times 10^{80}$$

Jawaban : C

B. Essay

1. $\frac{\sin^2 x - \sin^2 y}{\cos^2 x - \cos^2 y} = \frac{(1 - \cos^2 x) - (1 - \cos^2 y)}{\cos^2 x - \cos^2 y} = -1$

2. Andaikan $n < 1900$

$$\text{maka } n + S(n) \leq 1899 + 27 = 1926$$

jadi pasti $n \geq 1900$

- untuk $1900 \leq n \leq 1999$

Misalkan $n = 19ab$ dengan $0 \leq a, b \leq 9$ maka $n + S(n) = 1900 + 10a + b + 1 + 9 + a + b = 2016$

$$1910 + 11a + 2b = 2016$$

$$11a + 2b = 106$$

Mungkin saat $b = 9$ dan $a = 8$

maka $n = 1989$

- untuk $2000 \leq n \leq 2016$

andaikan $n = 20ab$

dengan $0 \leq a \leq 1$ dan $0 \leq b \leq 9$ (saat $a=1$, $0 \leq b \leq 6$)

maka $n + S(n) = 2016$

$$2000 + 10a + b + 2 + a + b = 2016$$

$$11a + 2b = 14$$

yang memenuhi hanya $a = 0$ dan $b = 7$

jadi $n = 2007$

$$x \left(\frac{2x^2}{z} \right) = 2z^2$$

$$2x^3 = 2z^3$$

$$x = z$$

3. $x^4 + 4y^4 = p$

$$x^4 + (2y^2)^2 = p$$

$$(x^2 + 2y^2)^2 - 2x^2(2y^2) = p$$

$$(x^2 + 2y^2)^2 - (2xy)^2 = p$$

$$(x^2 - 2xy + 2y^2)^2(x^2 + 2xy + 2y^2) = p$$

karena p hanya mempunyai 2 faktor yakni p dan 1 maka:

$$x^2 + 2xy + 2y^2 = p$$

$$x^2 - 2xy + 2y^2 = 1$$

$$(x-y)^2 + y^2 = 1$$

sehingga $p = 5$

banyaknya solusi (x,y) adalah $(1,1)$

maka : $\frac{z}{4x} = \frac{z}{4z} = \frac{1}{4}$

4. $a^2 + 2b^2 = 2ab$

$$(a^2 + 2b^2)^2 = (2ab)^2$$

$$a^4 + 4a^2b^2 + 4b^4 = 4a^2b^2$$

$$a^4 + 4b^4 = 0$$

5. $\frac{y}{8z} = \frac{z}{4x}$

$$xy = 2z^2 \dots(1)$$

$$\frac{x}{2y} = \frac{z}{4x}$$

$$yz = 2x^2$$

$$y = \frac{2x^2}{z} \dots\dots(2)$$

$$xy = 2z^2$$